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# Wave Exciting Force on A floating Rectangular Barge Due to Surface Waves

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**Abstract:** The prediction of parameters in sea keeping, for example, body response, wave load, deck wetness, slamming, among others, are some of the most important aspects in ship design. Furthermore, peak loads created by cyclone winds and waves and fatigue loads generated by waves over the body are also important design consideration for offshore construction. Offshore platforms which is part of offshore construction have many uses including oil exploration and production, navigation, ship loading and unloading and to support bridges and causeways, where offshore oil drilling is one of the most visible and imperative application. The structures aforementioned must function properly and safely for a longer period although they are subject to harsh marine environment. Due to these visible applications this paper is aimed at addressing impulsive hydrodynamic forces that come about due to presence of a rectangular floating barge on the wave terrain. The panel method developed by Hess and Smith (1964) will be used together with the appropriate Greens functions to help in deriving the integral equations.

**Index Terms:** Rectangular Floating Barge, Froude-Krylov Force, Diffraction Force, Wave Exciting Force, Panel Method, Heave wave exciting force, surge wave exciting forces.

## 1 INTRODUCTION

The problem of determining the wave exciting forces acting on a rigid floating body present on the wave terrain has been studied extensively over the years. These forces are associated with surface waves generated by: the interaction between winds and the sea surface, moving vessels, hurricanes, seismic disturbances or the gravitational pull of the sun and the moon.

Many researchers who have studied the wave exciting forces on rectangular floating bodies always assumed that the density of the fluid is not constant due to factors such as salinity and temperature change on the ocean [11]. However in classical view of ocean hydrodynamics, the fluid density is assumed constant [8], therefore, this paper aims at analysing the wave exciting forces with view that the fluid density is constant and of finite depth.

When an offshore body encounters surface waves it responds to these waves in six degrees of freedom. Translatory motions that is the heave, surge and sway and the oscillatory angular motions about the same axis, referred to as yaw, roll and pitch respectively [4]. However, the heave motion is the most influential of all of the six forces. For instance, low heave motion help in ensuring that the amount of deep sea mineral drilled in at a time is more. Furthermore, in case of ships the information on heave motion can be used to choose optimum ship routes based on relevant criteria like minimum fuel consumption or the shortest time of voyage.

## 2 Mathematical formulation

Three-dimensional problem concerning the hydrodynamic behaviour of a rectangular floating barge in the coastal marine environment is considered. The floating body is taken at a distance $h$ from the free surface, furthermore, it is assumed to have zero forward speed. The problem is analyzed in response to incoming regular waves with small amplitude $A$ as compared to the wavelength $\lambda$. The sea environment consists of a water layer of finite depth bounded above by the free surface and below by a rigid bottom. A fixed Cartesian coordinate $(x, y, z)$ is introduced with its origin on some point on the mean water level which is taken to coincide with the centre of floatation of the rigid floating body, and the $y-axis$ pointing upward through the centre of gravity of the body as shown in figure 1. In order to evaluate the physical problem, the fluid is assumed to be homogeneous, inviscid and incompressible and that the flow is irrotational.

## 3 Incident wave potential

With the assumption of the fluid being inviscid, incompressible and the flow being irrotational then velocity potentials in the fluid domain must satisfy the Laplace equations,

$$\nabla^2\phi = 0 \tag{1}$$

The linearized boundary conditions on the free surface that is, the dynamic free surface condition and the kinematic free surface condition together with the sea bottom conditions must also be satisfied.

$$\frac{\partial^2\phi_I}{\partial t^2} + g\frac{\partial\phi_I}{\partial y} = 0, y = 0 \tag{2}$$

$$\left(\frac{\partial\phi}{\partial t}\right)_{y=0} + g\eta = 0 \tag{3}$$

$$\frac{\partial\eta}{\partial t} = \left(\frac{\partial\phi}{\partial y}\right)_{y=0} \tag{4}$$

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The incident velocity potential that satisfies the above boundary conditions is defined as;

$$\phi_I(x,z,y,t) = \mathrm{Re}\left[\frac{ag}{\omega}\frac{\cosh k(y+h)}{\cosh kh}e^{ik(x\cos\theta + z\sin\theta - \omega t)}\right] \quad (5)$$

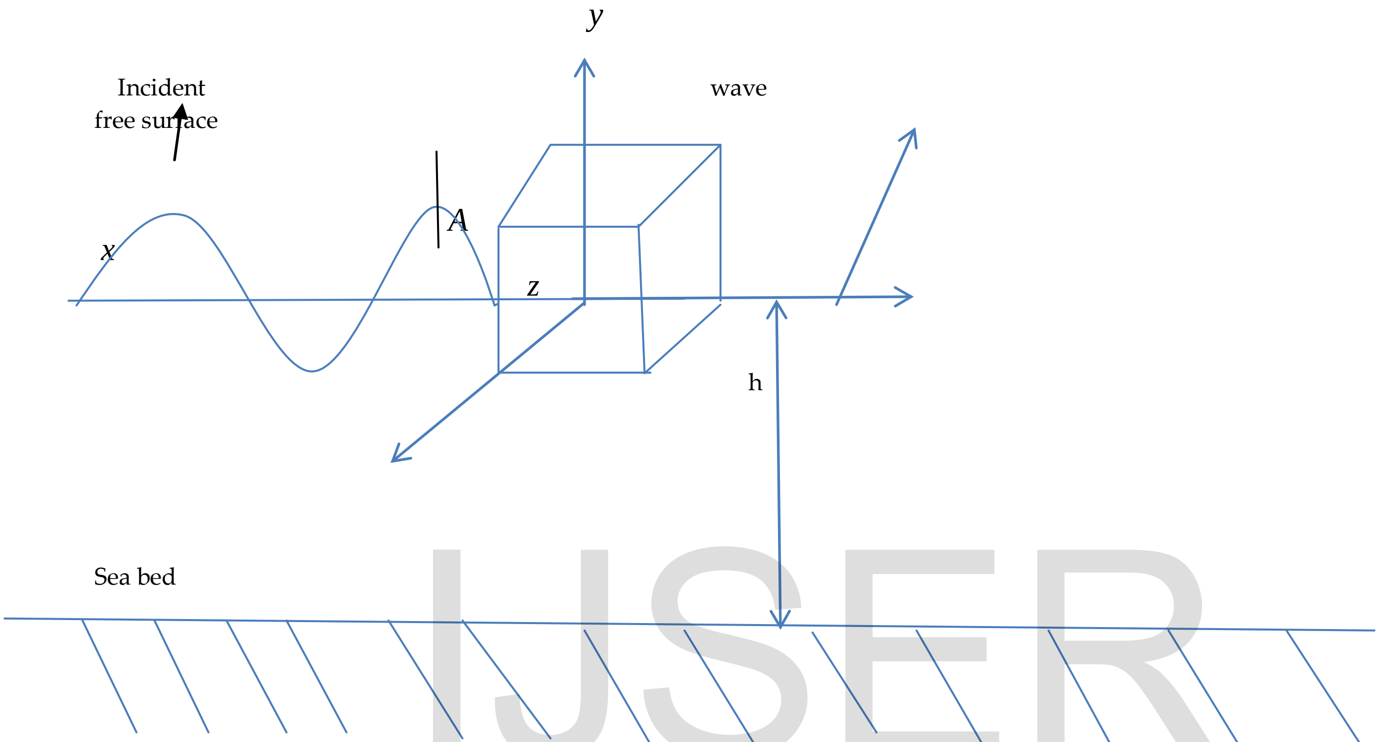


**Figure 1: Schematic diagram representing rectangular floating body on the incident wave field.**

## 4 Diffraction problem

Generally, linear theory can be applied to diffraction problem of any large offshore floating structure. Consequently, the boundary conditions are linearized. If the fluid surrounding the rectangular floating barge is assumed inviscid, incompressible and irrotational then, the potential function Laplace equation given by equation (6) can be applied to find the potential $\phi$.

$$\nabla^2\phi = \nabla\left(\frac{\partial^2\phi}{\partial x} + \frac{\partial^2\phi}{\partial z} + \frac{\partial^2\phi}{\partial y}\right) \quad (6)$$

$$\phi = \phi(x,z,y,t) = \phi_I(x,z,y,t) + \phi_D(x,z,y,t) \quad (7)$$

Where, $\phi_I, \phi_D$ are the velocity potential of the incident wave and diffraction wave respectively. In addition these velocity potentials must satisfy the following boundary conditions as aforementioned.

1. Free surface boundary conditions

$$\frac{\partial\phi}{\partial y} = \frac{\omega^2}{g}\phi, \text{ at } y = 0 \quad (8)$$

2. Bottom boundary condition

$$\frac{\partial\phi}{\partial y} = 0, \text{ at } y = -h \quad (9)$$

3. $$\frac{\partial\phi_I}{\partial n} = \vec{n}\bullet\vec{\nabla}\phi_I \quad (10)$$

4. The radiation boundary condition

$$\lim_{R\to\infty}\sqrt{R}\left(\frac{\partial\phi}{\partial n} - ik\phi\right) = 0 \quad (11)$$

## 5 Wave exciting force

Behaviours of offshore structures operating in the ocean environment are influenced by various kinds of external loads such as incident waves, thermal change, traffic loads, currents, and buoyancy. For any structure to be designed properly, the hydrodynamics loads must be predicted accurately. Furthermore, vertical acceleration and the relative vertical motion between the floating body and the waves are very important responses. They have always been associated with slamming (impulse loads with high pressure peaks that occur between the ship and the water) [1]. One of the main reasons for studying fluid motion and waves on a floating or a rigid body is to study the forces and moments acting on the body, that is, the hydrodynamic loads (Hydrodynamic loads are the forces and moment caused by a fluid on an oscillating body). These loads as aforementioned are present due to many factors, especially due to wave excitation. In the analysis of wave exciting force, incident waves are the most influential and most important in the determination of hydrodynamic forces especially the diffraction and the Froude Krylov forces [12]. Froude Krylov is the force introduced by the unsteady pressure field generated by undisturbed waves while the diffraction is due to the floating body disturbing the waves.

When an offshore body encounters surface waves with amplitude $A$ and direction $\theta$ incident upon the body, it moves with response to these waves in six degrees of freedom [2]. The resulting motion acting on these offshore structures according to linear theory will be very small. The corresponding velocity is sinusoidal and the corresponding frequency is equal to that of the incident waves. According to linear potential theory, the potential of a floating body can be expressed as a sum of the potential due the undisturbed incoming waves, the potential due to diffraction of the undisturbed incoming waves on the fixed body and the radiation potential due to the six body motions [10],[12], [16], [11].

$$\vec{F} = -\rho \operatorname{Re} i\omega A e^{i\omega t} \iint_{S_B} (\phi_I + \phi_D)\left(\frac{\partial \phi_i}{\partial n}\right) dS \tag{12}$$

$$\vec{F} = \iint_S \rho \frac{\partial \phi_I}{\partial t} \vec{n}\, dS + \iint_S \rho \frac{\partial \phi_D}{\partial t} \vec{n}\, dS \tag{13}$$

Equation (13) is the wave exciting force which is a combination of the Froude Krylov force and the diffraction force.

Then from equation (13) diffraction force is given by;

$$\vec{F}_D = \iint_S \rho \frac{\partial \phi_D}{\partial t} \vec{n}\, dS \tag{14}$$

But

$$\vec{n} = \frac{\partial \phi_k}{\partial n} \tag{15}$$

Where,

$$\frac{\partial \phi_k}{\partial n} = n_k, k = 1, 2, \ldots, 6 \tag{16}$$

$$n_1 = n_x, n_2 = n_z, n_3 = n_y \tag{17}$$

and

$$D = \begin{bmatrix} i & j & k \\ x & z & y \\ n_x & n_z & n_y \end{bmatrix} \tag{18}$$

$$= (zn_y - yn_z) i - (xn_y - yn_x) j + (xn_z - zn_x) k$$

Then from equation (18)

$$n_4 = zn_y - yn_z, n_5 = yn_x - xn_y, n_6 = xn_z - zn_x \tag{19}$$

Then substituting equation (15) on equation (14) we get

$$F = \iint_S \rho \frac{\partial \phi_D}{\partial t} \frac{\partial \phi_k}{\partial n} dS \tag{20}$$

$$F = i\rho\omega \iint_S \phi_D \frac{\partial \phi_k}{\partial n} dS$$

Then from equation (16) it can be seen clearly that equation (13) becomes

$$\vec{F}_D = i\rho\omega \iint_S \phi_k \frac{\partial \phi_D}{\partial n} dS \tag{21}$$

However, the incident velocity potential and the diffraction potential are related by the following potential.

$$\frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n} \tag{22}$$

Substituting equation (22) in equation (21) the wave exciting forces becomes;

$$\vec{F}_D = -i\rho\omega \iint_S \phi_k \frac{\partial \phi_I}{\partial n} dS, k = 1, 2, \ldots, 6 \tag{23}$$

From equation (23) we can derive the Froude Krylov force. The evaluation of Froude Krylov force commenced with work of Froude (1861) and Krylov (1896) and it is very important in the analysis of wave exciting force. The Froude-Krylov force is related to the incident wave potential and therefore from (23) we have

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$$F_I = -\rho \iint_S \frac{\partial \phi_I}{\partial t}\left(-\vec{n}\right) dS \quad (24)$$

$$\frac{\partial \phi_I}{\partial t} = \frac{\omega^2 \phi_I}{g} = i\omega \phi_I \quad (25)$$

Consequently, substituting equation (24) in equation (25) the wave diffraction force becomes,

$$F_I = \iint_S i\rho\omega\phi_I \vec{n} dS$$
$$= i\omega\rho \iint_S \phi_I \frac{\partial \phi_k}{\partial n} dS \quad (26)$$

However, the wave exciting force acting on a body is a combination of the Froude Krylov force and the diffraction force, hence combining equation (23) and (26) we get the wave exciting force as,

$$F_I + F_D = -i\omega\rho \iint_S \left(\phi_I + \phi_D\right) n_i dS \quad (27)$$

$$= i\rho\omega \iint_S \left(\phi_I \frac{\partial \phi_k}{\partial n} - \phi_k \frac{\partial \phi_I}{\partial n}\right) dS$$

## 6 Numerical analysis

Numerical methods for assessing the sea keeping of offshore structures to predict their motions and loads in waves have been used over the years. There exist practical numerical tools based on three-dimensional analyses that predict linear wave-induced motions and loads on large volume structures at zero Froude number. Strip theory is still used widely [5], though this method only provides good results for the vertical responses. In addition, this theory is based on the assumptions of potential flow, slender body and small amplitude motions. Consequently, the theory is computationally efficient, but cannot give accurate hull pressure predictions [9]. Moreover, for horizontal wave responses, this method needs some modifications to improve the result. Due to the shortcomings of this theory, researchers have devised better numerical methods to understand the hydrodynamic loads acting on offshore structures. For instance, [17], used the eigen function expansion to study forces on a freely floating circular cylinder. The approach of eigen function expansion was extended by [3] to the case of a freely floating circular cylinder and a submerged cylinder resting on the seabed. However, this method has considerable degrees of limitation for instance it is insufficient to model structures requiring a very large number of modes, which limits the size of the cross-section for 3-dimensional problems. Boundary Element Method (BEM) and Finite Element Method (FEM) have also been used in the analysis of the hydrodynamics loads of floating structures on the wave terrain. BEM and FEM are efficient in the numerical analysis of hydrodynamics, unfortunately, they give rise to fully populated matrices hence the computation time tend to grow depending on the size of the problem leading to complex computational procedures [5]. Therefore, to predict properly the interactions between different sections of the floating structure with waves, three-dimension panel method, which is based on potential theory, was developed [7], over the years, it has proved to be more efficient especially in sea-keeping calculations and analysis [12].

Panel methods attempt to solve the Laplace equation in the fluid domain by distributing sources and dipoles on the body and, in some methods, on the free surface. The surfaces are divided into panels, each one associated with a source and dipole distribution of unknown strength, [9]. The boundary conditions to be applied to the problem are often linearized and they determine either the potential or the normal velocity on each panel. Green's theorem is used to relate the source and dipole distribution strength to the potential and normal velocity on each panel. The number of panels plays a vital role in representing the shape of the body more accurately. Large number of panels ensures more accuracy on the shape of the body and hence more accurate results. In each panel the potential is taken to be constant. This method has advantages because it helps in reducing the dimensionality of the problem by one and helps in transforming an infinite domain of interest to finite boundaries in which the far field condition is automatically satisfied.

Due to the shortcomings of the outlined numerical Technique the Panel method developed by Hess and Smith (1964) has been used to obtain the radiation potentials and the diffraction potential which are very paramount in the calculation of the diffraction force and the Froude- Krylov force.

## 7 Results and discussion

The wave exciting forces is computed for a barge, the barge is modelled like a rectangular box. The measurement of the box are taken as those of Endo 1987 this is aimed to helping in the comparison of the results. The box has length $L$ and beam $B$ each of $90m$, and its draft D as $40m$. The body surface is defined by a set of points which are presumably exactly on the surface. These points are associated in a group of four to form quadrilateral surface elements. Each point on the surface is used in the formation of four quadrilaterals, and thus the total number of quadrilateral is approximately the same as the number of points used to define the body surface. The integration on each panel is carried out by the method of Hess and Smith (1964), $192$ Panel are used. In this case, the velocity potential is assumed constant over the panel. The boundary integral equations for radiation potentials are derived for the rectangular floating barge. These radiation potentials are used to solve diffraction forces and the Froude-Krylov forces for the rectangular floating barge on fluid of finite depth. After writing a Fortran code containing all the information above and the mathematical formulation aforementioned, the graph

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in fig 2, fig 3, fig 4 and fig 5 are obtained. The graphs are in agreement with others that have been obtained by other researches [12]; Endo, 1987). Fig 3 shows the magnitude of heave wave excitation which is very crucial to ocean engineers. The magnitude of the wave excitation force on the heave direction is inversely proportion to the wave frequency. In addition applying the dispersion relation $\omega^2 = gk \tanh kh$ [12] it shows that water depth has significant effect as far as heave motion is concerned. It is clear that if the depth is more then, the frequency will also be high, as clearly shown in fig 5. This is in accordance with shallow water effect. It is clear from the figure that in shallow fluids the heave exciting forces is very high. This explain why waves generated by seismic disturbances are very catastrophic compared to others surface gravity. Consequently, any offshore body will thrive well when the sea is deeper. From this result we are able to understand why the harbours are always constructed at very deep point near the shore. Moreover, for any offshore body to operate efficiently and safely there is need for low heave motion then from the results this can only be so when the wave frequency is very high. As aforementioned Fig 3 which shows the magnitude of the heave wave exciting force is in agreement with other results that have been used by other researches [12] and Endo (1987). This study establishes that the use of Green Method developed by [17] in its series form together with the Hess and Smith (1964) panel method are not only computationally simple but also very accurate in the analysis of the hydrodynamic loads.

From fig 3 it is evident that the magnitude of surge wave exciting forces increases as the wavelength increases up to a certain level where then it starts to decrease. It is worth noting that the surge wave exciting force does not have a lot of impact on offshore bodies and this explain why it has not be given a lot of attention. However, [12] analysed the magnitude of the surge wave in two layer fluid.

From fig 4 it is evident that change in water depth has a diverse effect on the magnitude of the wave exciting force. As the depth decreases the surge wave exciting force increases.

**CONCLUSION**

It is clear that Hess and Smith (1964) method which was employed to investigate the characteristic of floating body motion in finite fluid produces results which are in excellent agreement with experimental result. The green functions applied in this study is in series form hence it reduces the singularity on the free surface unlike the integral green function used by Endo (1987).

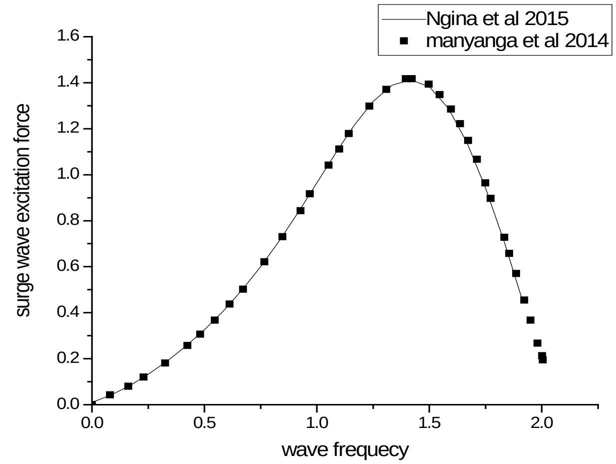


Figure 2: magnitude of surge wave exciting forces

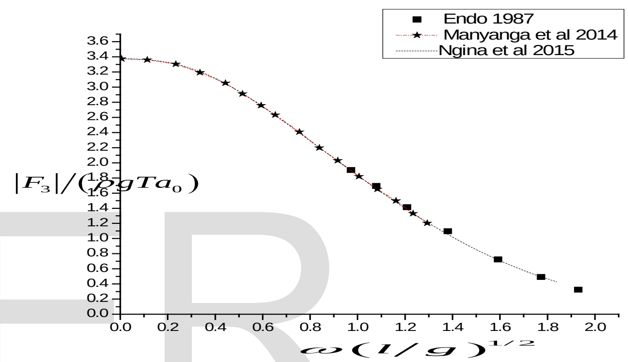


Figure 3: A graph showing the magnitude of heave wave exciting force

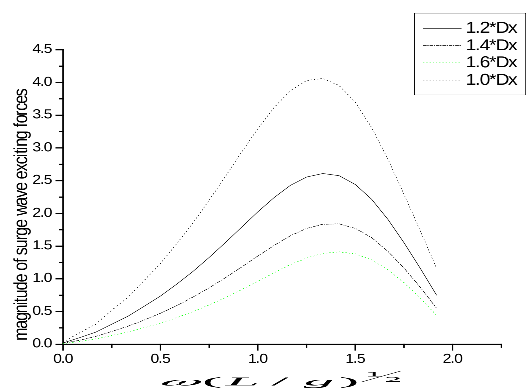


Figure 4: The magnitude of surge wave excitation force in different water depth

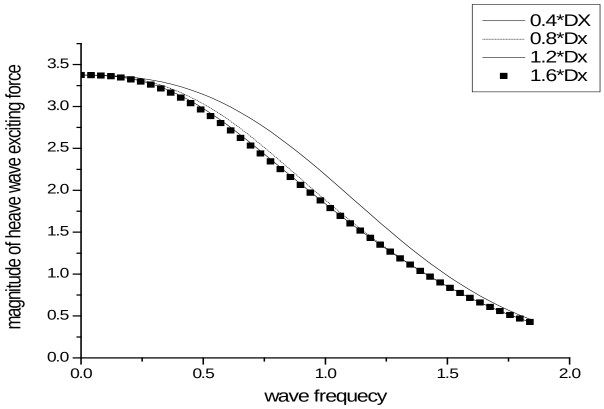


*Figure 5: Magnitude of heave wave excitation forces in varying depth*

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